Moment symbolic calculus in mathematical statistics

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Abstract

In the last ten years, the employment of symbolic methods has substantially extended both the theory and the applications of statistics and probability. This contribution [1] reviews the development of a symbolic technique arising from classical umbral calculus, as introduced by Rota and Taylor in 1994 [3]. The usefulness of this symbolic technique is twofold. The first is to show how new algebraic identities drive in discovering insights among topics apparently very far from each other and related to probability and statistics. One of the main tools is a formal generalization of the convolution of identical probability distributions, which allows us to employ compound Poisson random variables in various topics that are only somewhat interrelated. Having got a different and deeper viewpoint, the second goal is to show how to set up algorithmic processes performing efficiently algebraic calculations. Some of the algorithms have been implemented in `R` [2]. In particular, the challenge of finding these symbolic procedures should lead to new methods, and it poses new problems involving both computational and conceptual issues. Evidence of efficiency in applying this symbolic method will be shown within statistical inference, parameter estimation and, more generally, problems involving multivariate functions. Recent connections within random matrices have extended the applications of the symbolic method.

Keywords

Umbral calculus, composition of formal power series, cumulants, symmetric polynomials.

References

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