Fast and accurate calculations of gamma difference distribution characteristics

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Abstract

The distribution of the difference of two independent gamma random variables, in general with different shape parameters, is called gamma difference distribution (GDD).

It can be found in many recent applications such as controlling a measurement accuracy of optical detectors [3], detecting radar sensor threshold [7], setting optimal performance of wifi networks [4], chemotherapy cancer treatment [8], detecting eye glaucoma [1], etc.

Despite the wide applicability of GDD, the fast and precise calculation of particular values of probability density function and cumulative distribution function with unequal GDD shape parameters is still quite challenging. However, the need for accurate, reliable, numerically stable, and fast computations arises naturally in any real data analysis or computational research (e.g. Monte Carlo or bootstrap methods).

Therefore, in this work we investigate the four principal computational ways for GDD in the plethora of currently available computing tools. We pay special attention to open digital tools based on programming languages Python and R, which became significant during the last decade, in the light of enormous advances of open data science. Our focus is on GDD with unequal shape parameters resulting from time series kriging [2], a forecasting approach based on the best linear unbiased prediction and linear mixed models.

We proposed a combination of numerical inversion of the characteristic function [9] and the trapezoidal rule with the double exponential oscillatory transformation (DE quadrature) [6] and we implemented our open-source tool in high-performance Python (with Numba). The results of our numerical study, with emphasis on using open data science tools, demonstrate that it is exponentially fast, highly accurate, and very reliable.

Our approach could be potentially used in the future, e.g. for distributions expressible by linear combinations of characteristic functions in fields such as multidimensional statistics, measurement uncertainty analysis in metrology as well as in financial mathematics and risk analysis.

Keywords

Numerical inversion of the characteristic function, double exponential quadrature, computational tools, high-performance Python, econometrics, time series, kriging.

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