On multimodal distributions. Estimation methods and examples of applications in socio–economic geography

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Abstract

The distribution modality may be unimodal or bimodal, less often multimodal, depending on the frequency of the values of the analyzed random variable. In his classification system (AJUS) for distributions, Galting [3] distinguished two following types of multimodal distributions: U – bimodal, with peaks at both ends, and S - bimodal or multimodal, with multiple peaks. The problem of the statistical analysis of phenomena for which the empirical distributions could suggest departure from unimodality was noticed already at the end of the nineteenth century, when theoretical bases for statistics began to be constructed. Pearson [9] was the first who in 1894 proposed the procedure of the dissection of abnormal frequency-curves into normal curves. From then, during the next several decades, bimodal (also multimodal) distributions were estimated mainly via the mixture of normal distributions. The situation began to change with the development of computerization and the possibilities of numerical analyses of even very complicated statistical models. During the last twenty years, different bimodal and skew distributions have been proposed. The aim of the paper is to expand the idea proposed by [1] during Linstat2018 conference (based on a specific exponential distribution) concerning U and S type of distributions and compare with the results of other authors (as [4], [5], [6], [11]) taking into account some empirical applications of such models (e.g. in [2], [7], [8], [10], [12]), including applications in socio–economic geography.

Keywords

Multimodal distribution, Bimodal distribution, Exponential distribution.

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