Optimality of block designs under a hub correlation structure

Razieh Khodsiani[1] **and Saeid Pooladsaz**[2]

[1]*K. N. Toosi University of Technology, Tehran, Iran*
[2]*Isfahan University of Technology, Iran*

Abstract

Block experiments have been widely used in sciences, medicine and engineering. Some correlation structures can be considered in block experiments. The observations in different blocks are usually assumed to be uncorrelated but that observations within blocks are correlated with the same correlation structure.

The hub correlation is a special correlation structure with applications to experiments in genetics, networks and other areas in industry and agriculture. There is a known correlation between a hub plot (typically the first plot) and each of other plots (see Zhu [5]; Hero and Rajaratnam [3]). In general, the correlation matrix of each block for hub structure is as below,

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & \boldsymbol{\rho}'_{k-1} \\ \boldsymbol{\rho}_{k-1} & \mathbf{I}_{k-1} \end{bmatrix} \tag{1}$$

where $\boldsymbol{\rho}'_{k-1} = (\rho_2, \rho_3, \ldots, \rho_k)$, and ρ_i is the correlation between plots 1 and i, $i = 2, \ldots, k$. Note that there is no correlation between the plots $2, 3, \ldots, k$. Moreover, one can typically assume that the correlation between the first and i-th plots decays as i increases. For example, Hardin et al. [2] considered the case that $\rho_2, \rho_3, \ldots, \rho_k$ decrease from $\rho_{\max}$ to $\rho_{\min}$.

Some researchers have obtained the optimal designs under hub correlation structure with $\rho_i = \rho$, $i = 2, \ldots, k$. For example see Chang and Coster [1] and Khodsiani and Pooladsaz [4].

In a block experiment with hub correlation, if one of the correlation values in $\boldsymbol{\rho}_{k-1}$, say $\rho_{i'}$, depends on some elements such as location of i'-th plot in a block or the distance between the first and i'-th plots, then $\rho_{i'}$ may be different from ρ_i's, $i \neq i'$. So, we define $\mathbf{\Lambda}_a^{(i')}$ as the correlation matrix in (1) such that for specified plot i', $\rho_{i'} = \gamma\rho$, $0 \leq \gamma < 1$, and $\rho_i = \rho$ for all $i \neq i'$. In this presentation, by a decomposition of concurrence matrix of designs, we theoretically obtain the universally optimal block designs, binary or non-binary, under $\mathbf{\Lambda}_a^{(i')}$ for every i'.

Keywords

Universal optimality, hub correlation structure, generalized binary block design, Balanced block design.

References

[1] Chang, Li and Coster, D. C. (2015). Construction of weak universal optimal block designs with various correlation structures and block sizes. *Journal of Statistical Planning and Inference 160*, 1–10.

[2] Hardin, J., Garcia, S. R. and Golan, D. (2013). A method for generating realistic correlation matrices. *The Annals of Applied Statistics 7(3)*, 1733–1762.

[3] Hero, A. and Rajaratnam, B. (2012). Hub discovery in partial correlation graphs. *IEEE Transactions on Information Theory 58*, 6064–6078.

[4] Khodsiani, R. and Pooladsaz, S. (2017). Universal optimal block designs under hub correlation structure. *Statistics and Probability Letters 129*, 387–392.

[5] Zhu, Z. (2004). Optimal experimental designs with correlated observations, PhD thesis, Department of Mathematics and Statistics, Utah State University.