Controlling the bias for M-quantile estimators for small area

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Abstract

Representative outlier units occur frequently in surveys. As a result, several methods have been proposed to mitigate the effects of them in survey estimates. If outliers are a concern for estimation of population quantities, it is even more necessary to pay attention to them in a small area estimation (SAE) context, where sample size is usually very small and the estimation in often model-based. [3] explicitly addressed this issue of outlier robustness in SAE, using an approach based on fitting outlier robust M-quantile models to the survey data. More recently, [7] also addressed this issue from the perspective of linear mixed models. Both these approaches, however, use plug-in robust prediction, i.e. they replace parameter estimates in optimal, but outlier-sensitive, predictors by outlier robust versions (a robust-projective approach). Unfortunately, these predictors are efficient under correct model specification and assumptions, but they may be sensitive to the presence of outliers because they use plug-in robust prediction which usually leads to a low prediction variance and a considerable prediction bias. [5] and [4] proposed bias corrected method to reduce the prediction bias when the response variable is continuous. In this talk, we focus on M-quantile approach and we propose two general methods (i.e., for continuous and discrete data) to reduce the prediction bias of the robust M-quantile predictors in SAE context. The first estimator is based on the concept of conditional bias and extends the results of [1] and [6]. Then, we propose an unified approach to M-quantile predictors for continuous and discrete data which is based on a full bias correction and it could be viewed as a generalization of [2] approach. A Monte-Carlo simulation study is conducted and its results suggest that our approaches mainly improve the efficiency and they control the bias prediction error of M-quantile predictors when the population contains units that may be influential if selected in the sample.

Keywords

Small area estimation, Robust estimators, Bias correction.

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