On the ratio of extremal eigenvalues of β-Laguerre ensembles

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Abstract

Classical β-Laguerre ensembles consist of three special matrix models taking the form $\mathbf{X}\mathbf{X}^T$ with $\mathbf{X}$ denoting a random matrix having i.i.d. entries being real ($\beta = 1$), complex ($\beta = 2$) or quaternion ($\beta = 4$) normal distribution. It had been actually believed that no other choice of $\beta > 0$ (besides $1, 2$ and 4) would correspond to a matrix model $\mathbf{X}_\beta \mathbf{X}_\beta^T$ which can be constructed with entries from a classical distribution until the work [5]. Since then the spectral properties of general β-Laguerre ensembles have been extensively studied dealing with both the bulk case (involving all the eigenvalues) and the extremal case (addressing the (first few) largest and smallest eigenvalues). However, the ratio of the extremal eigenvalues (equivalently the condition number of $\mathbf{X}_\beta$) has not been well explored in the literature. In this paper, we study such ratio in terms of large deviations.

Keywords and phrases: β-Laguerre ensembles; extremal eigenvalues; large deviations
AMS 2010 subject classifications: 60B20, 60F10, 15A12

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